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Improved Motor Stability Predictions for 3-D Grains Using the SPP Code

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Nomenclature

A	Cross Sectional Area
C*	Characteristic velocity, $P_cA^*/\dot{m}$
M	Mach Number
$\dot{m}$	Mass flow rate
p	Pressure
r	nozzle radius
u	gas velocity
Y	Distance from the nozzle wall

Greek

α	Attenuation or growth constant
γ	Ratio of specific heats, Cp/Cv
ε	Expansion ratio (A_e/A^*)

subscripts

a	ambient
e	nozzle exit

superscripts

′	Fluctuation
*	Refers to nozzle throat plane
^	Acoustic quanity
-	Mean value

Abbreviations

ALPHA	α, Attenuation or growth constant
ALPHA DC	α_{dc}, Distributed combustion
ALPHA FT	α_{ft}, Flow turning
ALPHA NOZ	α_{noz}, Nozzle loss
ALPHA PART	α_{part}, Particle damping
ALPHA PL	α_{pl}, Pressure coupling
ALPHA VC	α_{vc}, Velocity coupling

Abstract

The JANNAF standard Axial Mode Instability Computer Program, SSP 1-D, has been linked to the three dimensional grain design module of the Solid Performance Program, SPP. The linkage of these two modules within the SPP allows solid propellant grain designers and analysts to automatically assess the impact of grain design changes on motor stability for a variety of web burn backs. Both the instability module and the three dimensional grain module are discussed and graphical representations for several motor grain surfaces are presented. Calculated results of total linear growth coefficient versus motor web are shown for two motors with three-dimensional grains.

Introduction

The SSP 1-D, Ref. 1, was first released by the Air Force Rocket Propulsion Laboratory for use by the solid rocket motor propulsion community in 1976 and received wide-spread distribution and usage. The usefulness of the program resulted in its incorporation in 1984 into the Solid Performance Program (SPP), Ref. 2, as a module.

The purpose of this work is to allow designers of solid propellant rocket motors to assess the impact of

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their design choices on motor stability during the preliminary design phase of the motor development cycle instead of waiting until a problem arises. Fixing a motor stability problem during the hot fire testing phase of the program is very expensive. Recent experience with the THAAD missile shows that while motor instability can be solved after it occurs, it is not currently designed out of missile systems before the propellant is cast. The computer program described here is the first step in adding to the rocket motor designers tool kit, a multi-dimensional capability for acoustics stability analysis which can be utilized as an adjunct to a grain design and internal ballistics analysis.

The work discussed here was part of a Small Business Innovative Research (SBIR) Phase I effort. The first part of this effort was to couple the existing 1-D SSP, (Ref. 1 and 3), code to the 3-D grain design module contained within the SPP, Ref. 4. This task would demonstrate that the proper linkage data could be transmitted from the 3-D grain design module to a stability code. The task was also intended to generate a useful tool for the rocket motor industry. The subject of this paper is to describe that tool.

The SSP Model

The Axial Mode Standard Stability Program, SSP-1D program is the standard computer program used in the United States to analyze solid rocket motors for linear longitudinal combustion instability. The computer program provides a standardized stability prediction method for solid rocket motors. Axial symmetry is assumed so that the real motor must be approximated by an axially symmetric motor. The method of analysis is fully described in Ref. 1 and 2. The method is applicable to rocket motors having arbitrary cross sectional flow area variation. The program is suitable for conducting parametric studies, and for assisting in the reduction of T-burner data. The program is based primarily on the linear theory of combustion instability. Thus linear stability elements consisting of pressure and velocity coupled driving, flow turning, wall losses, nozzle losses, residual combustion, and particulate damping are evaluated. The method of analysis is analytical. The mode shapes are evaluated in the frequency domain in terms of trigonometric and hyperbolic functions, and integration over the mode shapes yields the stability elements. Summation of the several gain and loss terms results in the growth rate of the pressure oscillations in the motor port. Since the linearized theory has been used, a linear growth rate is obtained, and this pertains only to the initially low amplitude portion of the period during which the pressure waves exhibit growth.

The SSP l-D computer program analyzes motor stability by first calculating the frequency of an acoustic mode. With the modal frequency available, the program evaluates pressure and velocity coupled gains, flow turning, nozzle losses, wall losses, residual combustion, and particulate damping. Amplitude dependent velocity coupled driving and subsequent mean pressure shifting is evaluated by a spectral decomposition of the effective erosive velocity. The growth constant, α, for each of these stability elements is computed and printed out, as is the total value. Plots of port area, acoustic pressure, acoustic velocity, and mean velocity vs. axial location are provided.

The frequency of the acoustic modes are those frequencies at which the boundary conditions are satisfied at both ends of the motor. A boundary condition of $\hat{u}=0$ at the head end of the motor is imposed, and at some trial frequency the condition at the aft end is calculated. If the boundary condition is not satisfied, the frequency is incremented by some fixed amount determined by input and the calculation is repeated.

Three boundary conditions at the aft end are allowed in the program: $\hat{u}=0$, $\hat{p}=0$, or the admittance boundary condition, $\hat{u}/\hat{p}-Y/\bar{\rho}a=0$. Depending upon which of the three conditions is specified in the input, the sign of the appropriate variable is checked for each trial frequency. When a change in sign is detected, a zero or a pole, of this variable has been crossed. A new estimate for the mode frequency is then made using the method of false position. Thus, once a zero or pole is crossed, it will always be found. This procedure is continued until the difference between successive trials is less than some desired tolerance. This procedure works well as long as the increment in trial frequencies is smaller than the distance between zeros. (Poles do not exist for the first two types of boundary conditions. Poles may exist for the admittance boundary condition.).

In the particle damping calculation, an integration is performed over the particle size distribution. Though an analytical expression for the integrand exists, an analytical expression for the integral has not been found. The integral is to be evaluated for diameters from zero to infinity. Since any numerical method must be over finite limits it was decided to use the "3-sigma" limits on the particle mass. Given a log-normal distribution in particle diameter, it is required to find an upper limit of integration that will assure that the integration covers essentially all of the particle mass. The interval so defined is then divided into 100 subintervals. Simpson's parabolic l/3 rule is used to evaluate the integral. The upper limit on particle diameter which assures that 99.87% of the particle mass is used.

In the mode coupling analysis a system of ten first-order ordinary differential equations are integrated with respect to time. The integration is performed using an Adams-Bashforth integration routine.

The SSP-1D program is contained as a module within a larger program called the Solid Performance Program, SPP. The SPP has the capability of computing the chemical equilibrium expansion of the motor exhaust, grain web burn back profiles, and motor ballistics. Results of these calculations are used to provide automated input to the stability analysis module, SSP l-D, so that a complete analysis at various grain burn backs can be performed automatically.

The SPP 3-D Grain DesignModule

The Three-Dimensional Grain Design Module (GDM) is based on the analysis of Peterson et al., Ref. 5. The geometry calculations performed by the GDM are based on the computation of cross sectional areas, then volumes, then changes in volumes, and finally surface areas. The method used simulates drafting techniques. The motor case is assumed full of propellant initially. Simulation is accomplished using five input figures in various combinations to describe the initial geometry. These figures include the frustum of a right circular cone, a right circular cylinder, a right triangular prism, a sphere, and a torus. These figures may overlap and/or protrude outside the case, and can be input as a out-burning void in the propellant, grain-filled in-burning figures, or non-burning solid figures. In addition, these figures may have rounded corners and edges. Each figure may be placed in any orientation in space, inside or outside of the grain. Ordinarily, a solid propellant grain is symmetric about the motor axis, so that only one sector of symmetry is analyzed.

Burning is assumed to be normal to the surface and the volume of propellant is calculated by integrating the propellant area along the grain axis. The propellant burning surface is obtained from the derivative of the volume with respect to the burn distance. The calculations are general in that any grain can be evaluated if the initial grain geometry is described using any combination of the five basic input figures. Since the analysis is formulated in an orthogonal coordinate system, auxiliary quantities, such as center of gravity and moment of inertia, are easily calculated.

Two types of figures may be input: 1) "Primitive" figures, which are the cone, cylinder, prism, sphere, and torus, and 2) "Macro" figures, which can be used to represent several different standard grain design types. Each Macro is composed of a set of Primitive figures. Macro figure input allows a standard grain design to be input from a drawing of the grain cross section shown in a typical set of blueprints. Currently, Macros are available for four of the most common geometry configurations, although new Macros can be added when needed. The four Macros currently available are: 1) Finocyl design, 2) Tapered Star design, 3) Simple Tapered Star design, and 4) Forward Star design. The methods used for combining primitive figures to obtain each of the Macro operations are described below. A more detailed description of the macros is given in Ref.5.

3-D Display Description

The matter of describing a complicated 3-D geometry is never simple. However, the pain of the process can be greatly reduced with a clever input scheme and good graphical diagnostic tools. The SPP 97 3-D grain design module of SPP has an adequate input scheme (see above) and reasonable 2-D graphics tools to help the user discover and correct input errors. Even so, it is sometimes difficult to figure out what has happened to cause an unexpected burn back profile. A 3-D graphical display of the burning surface was designed and implemented during this effort. The ability to display the propellant surface is crucial to this effort in that it also demonstrates that correct surface information can be generated from just knowing the cross sectional data at various points down the bore of the motor. Figure 1 shows a display of the initial and a burned back surface of the Extended Delta motor. Each characteristic of the motor design is clearly visible.

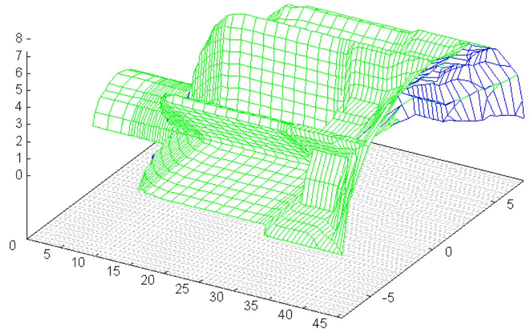

Figure 1a. Extended Delta Initial Burning Surface

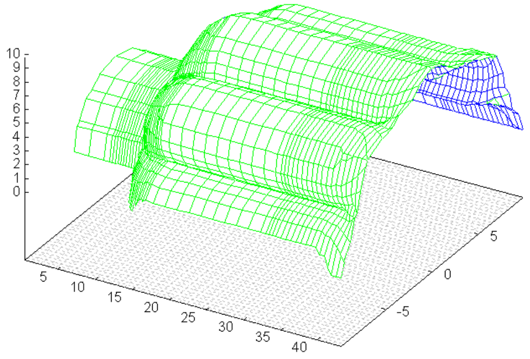

Figure 1b. Extended Delta Burning Surface
After 2.5 Inches of Burnback

The surface elements were generated from connecting adjacent cross-sectional data points. Degeneracies, that is quadrilaterals degenerating to triangles, were treated when points were removed from one cross section and not the next. The burning surface is viewed with the aid of a GUI driven Post Processor, GnuGen, and FreeWare software from Dartmouth University, GnuPlot[6], both are supplied with SPP97. The resulting plots represent the actual program solution, not a parallel solution. The figures above show the burning propellant surface at web burns of 0 and 2.5 inches. These figures show how the three dimensional effects dominate the solution. The significance of the three dimensional graphical capability becomes more apparent as the complexity of the motor increases.

Table 1 shows the required input to describe the Extended Delta grain geometry. The input value definitions are found in Refs. 2 and 4.

Table 1. Input For Extended Delta Grain Definition

```
SYMFAC=16,
 $END
SSTAR
 $IN XA=17.92, RTA=1.77233, RVA=7.2,
     XB=45.72, RTB=1.77233, RVB=7.2,
     ETA=27.1, R1=.25,
     MIRROR=T, $END
CYLINDER     CENTRAL PORT
 $IN C1=0,0,0, C2=46,0,0, R=1.88, $END
CYLINDER     FOR SUBMERGED NOZZLE
 $IN C1=34.22,0,0, C2=46,0,0, R=4.55, CR=.75, $END
CONE         FOR SUBMERGED NOZZLE
 $IN  C1=45.72,0,0,  C2=39.4,0,0,  R1=7.3,  R2=4.55,
$END
CYLINDER     AFT VOLUME
 $IN C1=45.7,0,0, C2=46,0,0, R=11.562, $END
FSTAR
 $IN XA=17.92, RTA=1.77233, RVA=7.2,
     ETA=27.1, R1=.25, R2=0., RDOME=7.2,
     MIRROR=T, $END
END FIGURES
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Verification

The SPP 7.0 3-D grain design module has been successfully linked to the axial mode instability module of SPP. The linkage has been tested thoroughly by comparing the instability results of 2-D and axisymmetric grain designs cases with those generated by the 3-D grain design module for the same grain configurations. Figure 2 shows a comparison between the stability results of a motor grain analyzed by the axisymmetric and 3-D grain design modules and then fed to the 1-D SSP. Four motor grains were looked at in this manner and the results of the comparisons were very good.

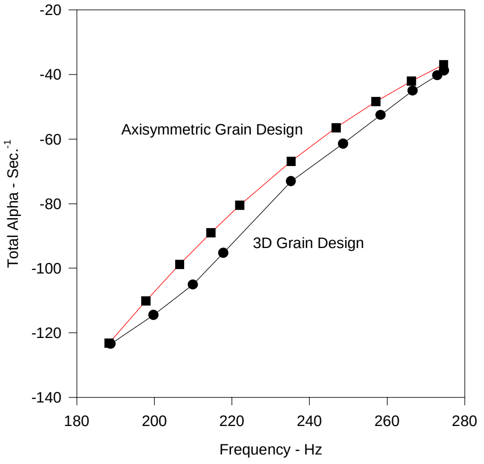

Figure 2. Comparisons of Stability Results
Using Different Grain Design Modules

The primary differences between the two computations stems from the treatment of vertical faces. In the axisymmetric grain design module, vertical faces can be readily identified and the mass addition from such faces are treated explicitly in the SSP1D. In the more general 3D grain design module, vertical faces are not explicitly tracked and hence the mass addition is treated the same as if it came from the cylindrical port by the SSP1D. However, the differences between the two calculations are much smaller than the accuracy of the method and the input data.

SSP 1-D Results With Three Dimensional Grains

The SSP 1-D was applied to several motors with 3-D grains. The first motor was the Space Shuttle ASRM motor. This motor is a multi-segmented motor with a 3-D finocyl in the forward segment as shown in Figure 3. below.

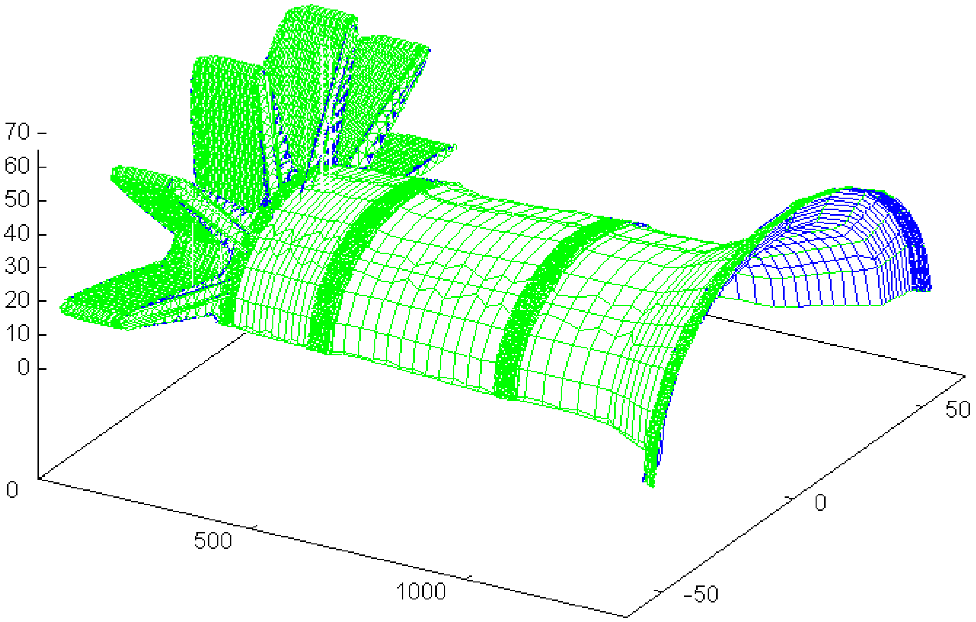

Figure 3. Space Shuttle ASRM Initial Grain Surface

The results of the calculations at several burn backs are shown in the Figure 4. below. The total growth coefficient is always negative and peaks near the end of the burn.

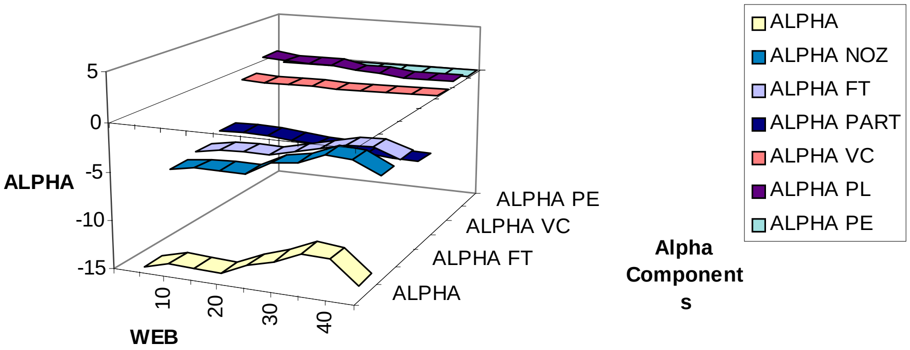

Figure 4. ASRM Axial Mode Results

In the absence of measured data on the propellant response function for this motor, the response function value was taken to be the burning rate exponent. Since this motor was never built, there is no comparison with measured data. However, experience with the shuttle SRM and RSRM would indicate that the agreement would be poor since the primary mechanism in these motors is vortex shedding which is not considered in the SSP 1-D.

The next motor consider is also a booster motor with and L/D of just over 4. A surface plot of the initial motor grain is shown in Figure 5.

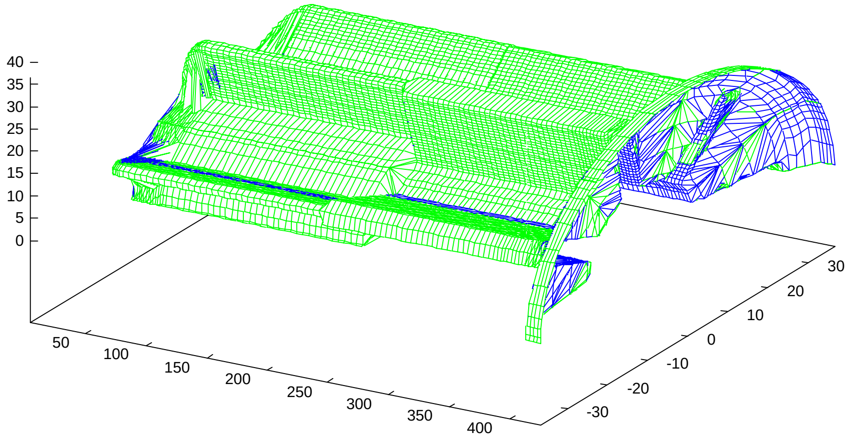

Figure 5. Short Booster Motor Initial Grain Geometry

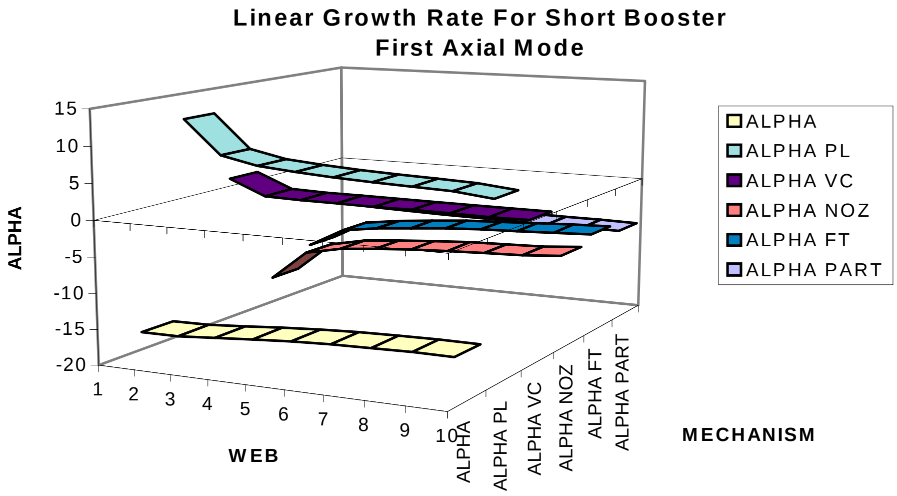

Figure 6 Linear Growth Rate Versus Web

The above figure indicates that the motor will be stable with the stability margin decreasing with web.

Conclusions And Recommendations

It is recommended that this work be extended to a three dimensional instability solver which would include the vorticity effects recommended by Flandro, Ref. 8, and the non-linear terms due to Culick, Ref. 9. The key to the proposed approach is to develop a method whereby the grain design information, i.e., grain surface or cross-sectional definition as a function of web, can be passed to a grid generation algorithm which will produce a grid of sufficient quality to provide an adequate descretization for numerically modeling the acoustic and flow fields within the motor cavity. With the motor cavity adequately described, the acoustic field can be modeled as a perturbation, Galerkin method, from the classical acoustic solution for the cavity. This approach is similar to the one used in the Multidimensional Standardized Stability Prediction (SSP 2-D) Program, Ref. 9.

References

1. Lovine, R.L. and Waugh, R.C., “Standardized Stability Prediction Method for Solid Rocket Motors”, Vol. I, Final Report, and Vol. III, Axial Mode Computer Program User’s Manual, Aerojet Solid Propulsion Company, AFRPL-TR-76-32, May 1976.

2. Nickerson, G.R., Coats, D.E., Hermsen, R.L., and Lamberty, J., A Computer Program for the Prediction of Solid Propellant Rocket Motor Performance (SPP), Vol. I, Final Report, by Software and Engineering Associates, Inc., AFRPL TR-83-036, September 1984.

3. Nickerson, G.R., Culick, F.E.C., and Dang, L.D., “Standard Stability Prediction Method for Solid Rocket Motors”, Air Force Rocket Propulsion Laboratory Report AFRPL TR-83-017, 1983.

4. Coats, D.E., and Dunn, S.S., “Solid Performance Program (SPP) Version 7.2 VAV”, SEA TR 95-02, Software and Engineering Associates, Inc., 1995

5. Peterson, E.G., Nielsen, C.C., Johnson, W.C., Cook, K. S., and Barron, J.G., "Generalized Coordinate Grain Design and Internal Ballistics Evaluation Program", AIAA Paper 68-490, CPIA/AIAA 3rd Solid Propulsion Conference, June 1968.

6. Williams, W., Kelley, C., et al., "GnuPlot Version 3.5, An Interactive Plotting Program", Email: info_gnuplot@dartmouth.edu, 1993

7. Flandro, G.A., “Effects of Vorticity on Rocket Combustion Stability”, **Journal of Propulsion and Power**, Vol. II, No. 4, July-August 1995.

8. Culick, F.E.C., "Nonlinear Behavior of Acoustic Waves in Combustion Chambers", Part I and II" Acta Astronautica, Vol 3 (1976) pp. 714-757.

9. Dudley, D.P. and Waugh, R.C., “Standardized Stability Prediction Methods for Solid Rocket Motors, Vol. II, Multidimensional Computer Program User’s Manual”, Aerojet Solid Propulsion Company, AFRPL-TR-76-32, May 1976.